☆ Kinetic Theory of gases

→ Perfect ~~of~~ or Ideal gas equation

A gas which fulfil the following condition is known as a perfect gas or an ideal gas. The molecules of the gas are point mass. The intermolecular force among the molecules of the gas is zero. The gas laws likes Charles law, ~~Bonarts~~ Boyles Law

Let us consider one mole of a perfect gas with its initial pressure P_1, Volume V_1, temperature T_1, respectively, P_2, V_2 T_2 with the final pressure, Volume, temp. of the gas respectively.
Let the initial state of the gas to the final state in two steps.

1. ~~Fe~~ Suppose that temp. T_1 of the gas is kept constant now the pressure of the gas is changed from P_1 to P_2 so that its volume change from V_1 to V_2. then acc. to

Boyl's law

$$P_1 V_1 = P_2 V_1'$$

$$\boxed{V_1' = \frac{P_1 V_1}{P_2}} - ①$$

Now suppose pressure V_2 is kept constant. Let the temp. of the gas change from T_1 to T_2 so that it's volume change from V_1' to V_2

Then, Acc. to Chale's Law

$$\frac{V_1'}{T_1} = \frac{V_2}{T_2}$$

$$\boxed{V_1' = V_2 \frac{T_1}{T_2}} - ②$$

from eq ① & ②

$$\frac{P_1 V_1}{P_2} = \frac{V_2 T_1}{T_2}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{PV}{T} = \text{Constant}$$

$$PV \propto T$$

$$\boxed{PV = RT}$$

$$\boxed{PV = \mu RT} - ③$$

Eq ③ is a perfect ideal gas equation. In eq ③

μ = no. of mole

R = universal gas Constant

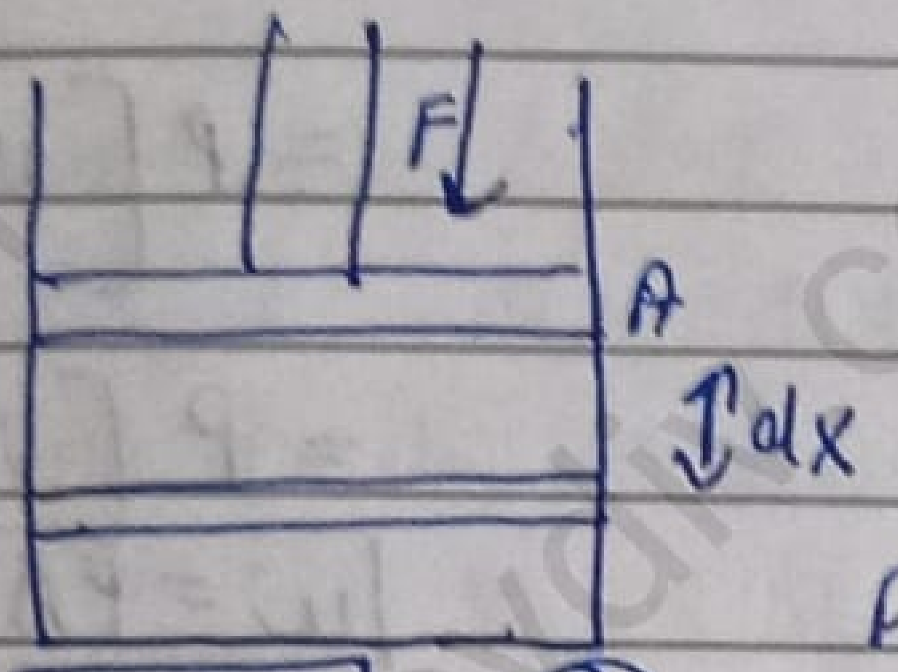
$R = 8.31\ J\ mol^{-1}\ K^{-1}$

Dimensional formula for $R = [ML^2T^{-2}K^{-1}mol^{-1}]$

Note:- Normal Temp. Pressure → 76 cm of Hg

↳ 273 K $\quad 1.013 \times 10^5 \ N/m^2$

Work done on a compressing gas



$P = \frac{F}{A}$

$\boxed{F = PA}$ — ①

$P_1, V_1, T_1 \rightarrow P_2, V_2, T_2$

$dw = Fdx \qquad dw = PA\,dx$

Let us consider a gas is filled in a cylinder and compressed due to this phenomenon dV be the small change in the volume of the gas then work done on the gas is given by

Required work done to small displacement dx

$$dw = Fdx$$

$$dW = PA\,dx$$

$dw = PA\,dx$

$dw = P\,dV$ — ②

because for Isothermal $PV = K$

$P = \frac{K}{V}$

Putting this value into ②

$dw = \frac{K}{V} dV$

$dw = RT \frac{1}{V} dV$

$\int_0^w dw = RT \int_{V_1}^{V_2} \frac{1}{V} dV$

$w = RT \left[\log_e V\right]_{V_1}^{V_2}$

$w = RT\left[\log_e V_2 - \log_e V_1\right]$

$w = RT \log_e \frac{V_2}{V_1}$

$w = 2.303\, RT \log_{10} \frac{V_2}{V_1}$

$w = 2.303\, RT \log_{10} \frac{P_1}{P_2}$

Note:- This is the special case for work done on compressing a gas if the volume of gas is changed at constant ~~temp~~ pressure then the total work done of gas is given by $W = P\int_{V_1}^{V_2} dV$

$$= P[V]_{V_1}^{V_2}$$

$$= P[V_2 - V_1]$$

$$\boxed{W = P\Delta V}$$

Imp. Assumption (Postulates) of Kinetic theory of gases

Kinetic theory is based on following assumptions:-

1. A gas Consist of a very large no. of molecules which are perfect elastic spheres, for a given gas they are identical in all respect but for diff. gases they are different.

2. The molecules do not have any ~~perfect~~ prefferred direction of motion but their motion is completely random.

3. They move in all direction with diff. speed 4. the side of the gas is very small (approx 2Å) 10^{-10} m and the average distance b/w the molecules is 20Å.

5. There is no concentration of molecules at any point inside the container.

6. The molecules don't exact any force of attraction or repulsion to each other.

⇒ **Most probable speed**

It is that speed which is passesed by maximum fraction of total no. of molecules of the gas $v_{mp} = \sqrt{\frac{2K_BT}{m}}$ (i)

Mean Average speed

Mean speed ar average speed is the average speed with which a molecule of the gas ~~from the ma~~ move

$$V_{av} = \frac{V_1 + V_2 + V_3 - - - . V_n}{n}$$

From the max wellian speed distribution law

$$V_{av} = \sqrt{\frac{8K_BT}{m\pi}} \quad -(ii)$$

3. **Root Mean square speed**

V_{rms} - Root mean square speed of a gas molecule is defined as the square root of the mean of squares of random velocities of the individual molecules of the gas

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 - - - + V_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3K_BT}{m}}$$

MCQ Imp

$$V_{mp} : V_{av} : V_{rms} = \sqrt{\frac{2K_BT}{m}} : \sqrt{\frac{8K_BT}{m\pi}} : \sqrt{\frac{3K_BT}{m}}$$

$$= \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$$

$K_BT = RT$

$V_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3RT}{m}}$

$V_{rms} = \sqrt{\frac{3RT}{m}}$

$V_{rms} \propto \sqrt{T} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

$V_{rms} \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{m_2}{m_1}}$ ~~(II)~~

Q Find out the ratio of V_{rms} speed for hydrogen and oxygen gases.

$$\frac{V_{H_2}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}}$$

$$= \sqrt{\frac{16}{1}} = 4:1$$

Degree of freedom

The no. of degree of freedom of a dynamical system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system

$$N = 3A - R$$

N ↓ No. of degree of freedom

$3A$ ↓ No. of particles of system

R ↳ Common Co-ordinates of the particles of the system

Gases	A	R	N=3A-2
· Monoatomic	1	0	3
· diatomic	2	1	5
· Triatomic linear	3	2	7
· Triatomic Non-linear	3	3	6

~~Kinetic interpretations of Temperature & Absolute Temp.~~

Pressure exerted by ~~or~~ ideal gas on walls of container is $P = \frac{1}{3}\frac{\overset{\text{Mass}}{M}}{\underset{\text{volume}}{V}} V_{rms}^2$

OR

$$P = \frac{1}{3}\overset{\text{density}}{\rho} V_{rms}^2$$

Kinetic Interpretations of Temperature & Absolute Temp.

Alternative method

Because we know that

$$V_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{—} ①$$

M = Total mass

$$M = N_A m \quad \text{—} ②$$

$$V_{rms}^2 = \frac{3RT}{M}$$

$M v_{rms}^2 = 3RT$

~~$\frac{1}{3}$~~ $\frac{1}{3} M v_{rms}^2 = RT$

from eq ②

$\frac{1}{3} N_A m v_{rms}^2 = RT$

$\frac{1}{3} m v_{rms}^2 = \frac{R}{N_A} T$

$\frac{1}{3} m v_{rms}^2 = K_B T$

$\frac{3}{2} \times \cancel{\frac{3}{2}} \cdot \cancel{\frac{1}{3}} m v_{rms}^2 = \frac{3}{2} K_B T$

$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} K_B T$

$\frac{1}{2} m v_{rms}^2 \propto T$ — ③

$$\left.\begin{array}{l} \text{if } T = 0 \\ m \neq 0 \\ v_{rm} = 0 \end{array}\right] \text{— ④}$$

Note:- N_A = Avogadro No.
K_B = Boltz Mann's constant
$K_B = 1.38 \times 10^{-23}$ J/mol.K

from the above eq ③ it is clear that temp. of a gas is directly propotional to the average translational Kinetic energy of the molecules of gas & from eq ④ it is clear that absolute 0 temp. is that temp. at which the molecular motion or molecules of the gas come to rest

law of equipartition of Energy

Acc. to this law for any system in thermal Equilibrium the total energy of the system is equally distributed among its various degree of freedom & the energy associated with each degree of freedom per molecule is equal to $\frac{1}{2} K_B \cancel{T} T$

Specific heat Capacity of gases

~~Gases~~ Gases	C_V	$C_P = C_V + R$
Monoatomic	$\frac{3}{2}R$	$\frac{5}{2}R$
diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$
Triatomic	$\frac{7}{2}R$	$\frac{9}{2}R$

C_V for monoatomic gas

degree of freedom for monoatomic = 3

Energy associated with each degree

$$= \frac{1}{2} K_B T$$

Total energy for 3 degree $\frac{3}{2} K_B T$ — ①

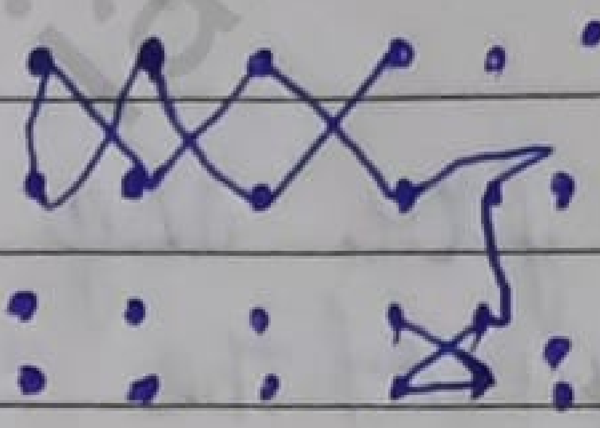
Total energy associated with 1 mole of gas

$$U = \frac{3}{2} N_A K_B T$$

$$U = \frac{3}{2} RT$$

$$C_V = \frac{d}{dT} U = \frac{d}{dT}\left[\frac{3}{2} R T\right] = \frac{3}{2} R \frac{dT}{dT} = \frac{3}{2} R$$

Mean Free path of a gas molecule is defined as the average distance travelld by the gas molecules b/w two successive collision.



If $\lambda_1, \lambda_2, \lambda_3 ------ \lambda_n$ be the distance travelled by the gas molecules during collision resp. then the mean free path ~~its~~ of the gas molecule is

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + --- \lambda_n}{n}$$

* Assumptions of mean free path

- The molecules of a gas are considered as hard spheres each of diameter t
- The collision between gas molecules are perfectly elastic
- The molecules of a gas except the molecules under consideration are at rest
- A molecule of a gas under consideration collide with all those molecules whose centres are at distance t from centre of molecules under consideration

Note :- Mean free path can be given in the terms of absolute terms, diameter (distance between two molecules) centres & pressure.

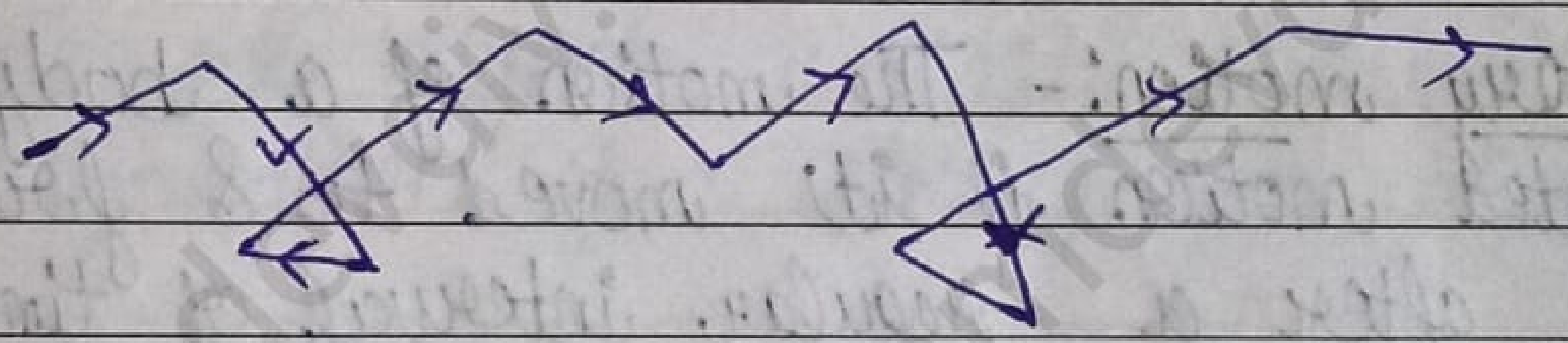
$$\lambda = \frac{K_B T}{\sqrt{2} \pi d^2 P}$$

→ Factor on which mean free path depend

- It is directly propotional to absolute temp 't' of the gas
- It is inversely propotinal to pressure P of the gas
- It is inversely propotial to square of the diamater of the gas malecule

Brownian Motion

A random or Zig-Zag motion of small particles suspected in a malecule is called Brownian Motion



→ factor affecting the Brownian Motion.

- Brownian Motion is directly propotional to the temp. of the medium
- Brownian Motion is inversely propotional to the viscocity of the Medium
- Brownian Motion is inversely propotial to the density of the medium
- Brownian Motion is inversely propotional to the the size of the particle